Traditional and New Keynesian Dynamic Models for Potential Output and Inflation Rate

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This paper uses the Kalman filter to estimate potential output as a latent process. We estimate two Dynamic Linear Models, comparing the results obtained through a traditional and a New Keynesian model. We verify that the traditional measures of output gap, even if usually applied in the estimation of the New Keynesian Phillips curve, are not consistent with the theory. We propose a New Keynesian measure that overcomes this limit. We suggest it as an alternative to the use of marginal costs. [JEL Classification: C32; E31; E32].

Keywords: Kalman filter; new Keynesian Phillips curve; output gap; unobserved component.

1. - Introduction

In this paper we study the relation between output and inflation, comparing the results obtained through a traditional and a New Keynesian analysis. We start from the difficulties that the New Keynesian economists usually meet in estimating the New

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Keynesian Phillips curve and argue that these difficulties are related to the use of inappropriate measures of output gap.

The dynamics of inflation implied by the two approaches are very different. While the traditional Phillips curve (PC) is *backward looking*, and states that present inflation depends directly on past inflation, the Phillips curve derived by the New Keynesian model (NKPC) is *forward looking*: inflation depends on past values only through the expectations. This difference is reflected in the theorethical cross correlation between inflation and output gap: the New Keynesian Phillips curve implies that inflation leads output, in the sense that an increase in inflation will signal a future increase in the output gap; but, empirically, using a traditional measure, the opposite pattern is found – output leads inflation – and this is better interpreted by the «old» Phillips curve. Galì and Gertler (1999), for example, estimated the empirical cross correlation between a traditional measure of output gap and inflation, noting an unilateral causality from output to inflation, in contrast with the New Keynesian theory.

As in Neiss and Nelson (2002), we suppose that the poor performance of NKPC in describing the dynamics of inflation depends on the unsuitability of traditional measures of output gap in a New Keynesian framework. This has been pointed out by many authors (see, for example, Galì, 2002). Traditionally, the output gap is estimated as the difference between observed output and a smooth measure of potential output. This method of estimating the output gap does not conform to the definition given in the New Keynesian literature. For example, Woodford says:

> *For according to our theoretical definition of the natural rate of output, Y_n should vary in response to real disturbances of any several types (productivity shocks, taste shocks of various sort, and variation in government purchases have already been discussed), and there is no* a priori *reason to suppose that these disturbances should be either small or properly described by smooth time series.*[1]

[1] WOODFORD M. (2003).

Our purpose is to verify Woodford's suggestion, stressing differences between the traditional interpretation of output gap and the New Keynesian interpretation. To do so, we compare two bivariate analyses involving output and inflation. To find a New Keynesian measure of output gap, we estimate it as a latent process, as in Kuttner (1994), using the Dynamic Linear Model (DLM) approach. The parameters of the different models are estimated by maximum likelihood, after which, the Kalman filter recursion is employed. Computations are implemented by the R package «dlm», available from the R-archive *www.R-project.org*.[2] In particular, if we describe the economy using the New Keynesian model, we find a new measure of output gap, and the relation between output and inflation is different from the one implied using traditional measures: the cross correlation between output gap and inflation is then coherent with the predictions of the theory.

Marginal costs can be used in the NKPC instead of output, as in Galì and Gertler (1999) and Sbordone (2002). The use of marginal costs is consistent with the theory, and the estimates of Galì and Gertler (1999) are correctly signed and well interpretable. As noticed also in Woodford (2001), marginal costs represent a good proxy for output gap, and are negatively correlated with a traditionally detrended output series. The method we propose is alternative to the use of marginal costs. Since output gap is an important variable in the implementation of monetary policy (see again Woodford, 2001) we estimate a theory-consistent measure based on the series of output.

We proceed as follows. In Section 2, with an univariate analysis, we try to interpret Woodford's suggestion focusing on the different patterns of potential output in relation to the economic interpretation of shocks. Section 3 presents a bivariate estimate of output gap and inflation designated as "traditional" because potential output moves smoothly, and inflation is described by a traditional PC. In Section 4, we present a bivariate estimate, called New Keynesian, where potential output is much more volatile and

[2] PETRIS G. (2006) and PETRIS G., PETRONE S. and CAMPAGNOLI P. (2009) explains how to implement the estimates with the package «dlm».

hybrid form of NKPC is used to estimate inflation. We compare these two models in Section 5 and conclude in Section 6.

2. - Univariate Models

In this section, we try to give an economic interpretation of the "degree of smoothness" of potential output. To do so, we analyze the behavior of potential output employing different assumptions about shocks.

To approach the problem more formally, we consider a general representation of a structural model for output reported in system (1),

$$\begin{aligned} &y_t = y_t^p + x_t && \\ &y_t^p = y_{t-1}^p + \delta_{t-1} + e_t && e_t \sim N(0,\sigma_e^2) \\ &\delta_t = \delta_{t-1} + z_t && z_t \sim N(0,\sigma_z^2) \\ &x_t = \phi(L)x_t + b + u_t && u_t \sim N(0,\sigma_u^2) \end{aligned} \tag{1}$$

where all variables are in logarithm. In particular, y_t is the output at time t, y_t^p is the potential output, x_t is the output gap and δ_t represents the growth rate of the potential. Following Harvey and Todd (1983), y_t^p is described with a linear growth model in the second and third equations. The last equation expresses the output gap as a stationary process, where $\phi(L)x_t$ is a polynomial in the lagged or leading values of x_t, and b is a vector of regressors. Finally, e_t, z_t and u_t are three random errors normally distributed with zero mean and variances, respectively, σ_e^2, σ_z^2 and σ_u^2.

One of the advantages of using a structural model lies in the opportunity it gives for an easy economic interpretation. In our case, the model described by system *(1)* is very useful because, on one hand it provides for the usual distinction between permanent and transitory shocks, and on the other hand it allows for different kinds of permanent shocks: e_t and z_t.

This feature is useful in examining different models. Forgetting the banal case of linear trend, where σ_e^2 and σ_z^2 are both equal

to zero, we consider three particular cases of equation *(1)*, studying the pattern of y_t^p when:
1) $\sigma_z^2 = 0$ and σ_e^2, $\sigma_u^2 \neq 0$, (Model 1), so that the growth rate of potential output δ is constant over time, and permanent shocks attain the level of y_t^p. In that case y_t^p is a random walk with a drift. This model, used (among others) by Watson (1986), Kuttner (1994) and Planas, Rossi and Fiorentini (2008), only allows for permanent shocks to the level of potential, disregarding shocks to the growth rate. As Watson (1986) pointed out, this simplification is justified by the assumption of Δy_t stationary.
2) $\sigma_e^2 = 0$ and σ_z^2, $\sigma_u^2 \neq 0$ (Model 2) implying that every permanent shock is of the second kind. Using this specification, we obtain a smooth measure of potential output, qualitatively similar to the one obtained in most of the econometric results concerning the natural rate of output. An authoritative example is provided by the analysis of CBO, and the famous Hodrick-Prescott filter is a particular case of this model.
3) σ_e^2, σ_z^2, $\sigma_u^2 \neq 0$ (Model 3), is the most general case analyzed by Harvey (1985) and Clark (1987).

In each model the output gap (x_t) is supposed to be a simple second-order autoregressive process, with the coefficients of the lagged values restricted to guarantee stationarity. This is a common specification in the literature.

Note that the models analyzed in this section have different econometric characteristics. In Model 1, the series of output is supposed to be integrated of order one while, in Model 2 and in Model 3, y_t is $I(2)$.

2.1 *Empirical Results*

We analyze the behavior of the US economy using quarterly data. More precisely, as a measure of output, we consider real GDP in billions of chained (2000) dollars, from 1950Q1 to 2004Q4, using data published by the Bureau of Economic Analysis.[3] Table

[3] Data are available at the web site: *research.stlouisfed.org/fred2/*.

1 reports the maximum likelihood estimate of the parameters. Tests on residuals (not reported) are made to check the validity of the models. Model 1 and Model 3 are very similar: in both cases about half of the variations of real output are attributed to transitory disturbances, while the other half are explained by permanent changes. On the other hand, Model 2 allocates nearly all the variations of output to the cyclical component. The potential output implied by Model 1 and Model 3 is quite close to that described by Woodford, of seeking a non-smooth measure of the natural rate of output, and this follows our reasoning about shocks: shocks to levels of growth imply sudden changes to potential output rendering it «irregular». Traditional measures of output gap, on the other hand, are obtained by a smooth estimate of potential output, and they imply (as shown by the estimates of the parameters in Model 2) that nearly all variation in output reflects transitory shocks.

TABLE 1

ESTIMATE OF PARAMETERS

	ϕ_1	ϕ_2	σ_e	σ_z	σ_u
Model 1	1.49	-0.55	0.0059		0.0061
[427.89]	(13.57)	(-4.86)	(2.62)		(2.37)
Model 2	0.99	-0.17		0.001	0.0084
[424.8]	(11.61)	(-13.08)		(23.4)	(22.9)
Model 3	1.48	-0.55	0.0058	0.00008	0.0061
[427.94]	(13.16)	(-4.78)	(2.45)	(0.3)	(2.35)

Likelihood in brackets, *t*-stat in parentheses.

When we consider disturbances both to the level and to the growth rate of potential, the latter seems to be less determinant in explaining the variation of output: σ_z^2 is very close to zero and statistically non significant. This last issue could suggest that it is not necessary to take into account variations in the growth rate

δ when shocks to the level are also considered. Peter Clark (1987), for example, found something similar in the analysis of GNP: «Allowing a variable growth rate for the trend component turns out to be inessential from a statistical point of view». A related concept is also expressed by Harvey (1985): «… since it appears that σ_ς^2 is quite small in practice, any conclusions reached under the assumption that it is zero are unlikely to be misleading». On the other hand, we can rebate stressing two points: a model with σ_z^2 different from zero helps to capture long period information about δ_t and it improves the likelihood. Moreover Model 3 is well interpretable in the New Keynesian framework, where all shocks but monetary ones hit the natural level of output. As Woodford proposed, we would like a natural rate that varies «in response to real disturbances of any several types». A potential output that responds to disturbances, to both the level and the growth rate, seems to be a good representation.[4]

It is appropriate to cite Clark (1987) again: «At least two avenues for distinguishing between the two primary alternatives (smooth trend plus large persistent cycles *versus* irregular trend plus small random cycle) remain to be explored». We have seen that different ways of interpreting shocks lead to different detrending methods, and to "the two primary alternatives". Our purpose will be to extend this comparison in a bivariate model for the output and the inflation rate, comparing the traditional and New Keynesian approaches.

3. - A Traditional Estimate of Output Gap and Inflation

The first bivariate model we analyze is considered "traditional" for the two reasons previously highlighted: the output gap is the difference between the observed output and a smooth poten-

[4] In principle we could conciliate a time varying growth rate of potential with the hypothesis that output is $I(1)$, describing δ_t as a stationary stochastic process. However we were not able to find a good estimate for such a model via maximum likelihood, because the likelihood function is, in this case, multimodal.

tial output; inflation is described by a backward looking Phillips curve. The model for output is a particular case of Model 2 (see the previous section) utilizing the popular Hodrick-Prescott (HP) filter (see Hodrick and Prescott, 1997). As in Harvey (2002), the filter is simply obtained imposing, in Model 2, $\phi_1 = \phi_2 = 0$, and the ratio $\frac{\sigma_z^2}{\sigma_u^2} = \frac{1}{\lambda}$ (remember that u_t is the transitory disturbance and z_t represents the shocks to the growth rate), where λ is the penalty parameter of HP. For quarterly data, λ is commonly set equal to 1600, as suggested by Hodrick and Prescott.

So, the "traditional model" (TM) analyzed in this section is:

$$
\begin{aligned}
& y_t = y_t^p + x_t \\
& y_t^p = y_{t-1}^p + \delta_{t-1} \\
& \delta_t = \delta_{t-1} + z_t && z_t \sim N(0, \sigma_z^2) \\
& x_t = u_t && u_t \sim N(0, \lambda\sigma_z^2) \\
& \pi_t = c + \alpha(L)\pi_t + kx_t + \varepsilon_t && \varepsilon_t \sim N(0, \lambda\sigma_\varepsilon^2)
\end{aligned}
\qquad \text{(TM)}
$$

where $\alpha(L)\pi_t$ is a polynomial in the lagged values of π_t.

3.1 *Data*

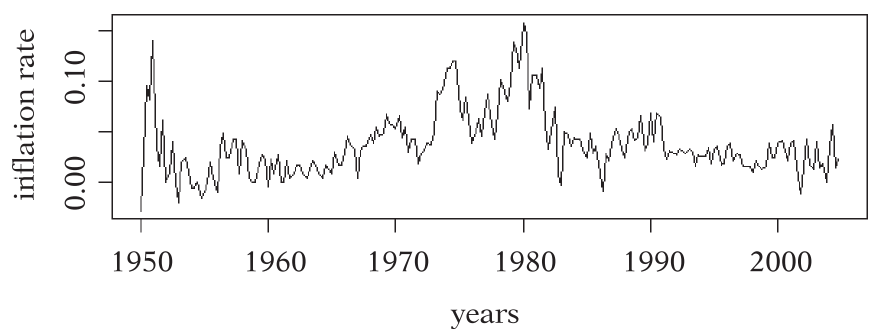
Inflation is measured quarterly as the (annualized) rate of change of the level of prices between two subsequent quarters. For the latter, we used the Consumer Price Index (all items) from the US Department of Labor: Bureau of Labor Statistics.[5] This series reports monthly data, therefore, we take the average level of price for each quarter. The choice of CPI, as noted by Neiss and Nelson (2002), is the most appropriate in this framework, being the CPI targeted by central banks. We express both inflation and the output gap in decimals, instead of in percentage points. The data are plotted in Graph 1.

[5] Data are available at the web site: *research.stlouisfed.org/fred2/*.

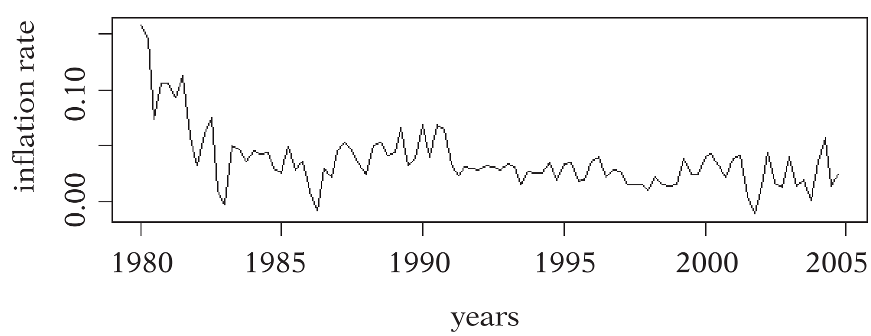
GRAPH 1

US QUARTERLY INFLATION RATE

1950Q1-2004Q4



1980Q1-2004Q4



In this section we consider a traditional specification for the Phillips curve where inflation is explained by an autoregressive process, and by the output gap that is supposed to be stationary. This implies that good estimates can be expected only if the inflation process is stationary. However, this is not the case for the entire sample we considered in the univariate analysis; the ADF test in Table 2 confirms the impression we have when looking at the plotted series in the first panel of Graph 1. A possible method would be to differentiate the series d times until stationarity is achieved, and then to estimate an ARMA model. Kuttner (1994) followed these steps, estimating a bivariate model for output and inflation. However, in doing so, we will lose any advantage, offered by the structural model approach, of a clear interpretation of the coefficients. An alternative solution consists in limiting our analysis to a stationary subsample. We chose to analyze the period starting with the first quarter of 1980 through the last quarter of 2004 (see Table 2 and the second panel of Graph 1).

TABLE 2

AUGMENTED DICKEY - FULLER TEST

data: inflation	
Sample	p-value
1950Q1-2004Q4	0.2229
1980Q1-2004Q4	0.02183

3.2 *Empirical Results*

We found that a Phillips curve with four lags is a good representation for inflation.

The estimates of the model's seven parameters are reported in Table 3. Even if not all of them are significantly different from zero, the residuals are not autocorrelated, and the Shapiro test (not reported) confirms that they are normally distributed. This tells us that the model is good for explaining the dynamics of both output and inflation. The constant c is considered as a latent process, and its estimate is 0.012.

As in the case of Model 2 in the previous section, most of the variation of output is explained by transitory shocks: σ_u is, by construction, 1,600 times greater than the value of σ_z.

As we have seen, the HP filter is a particular case of Model 2, and, therefore, the permanent shocks considered are only those to the growth rate of output generating a measure of potential output very smooth, see Graph 2. The output gap, shown in Graph

Table 3

ESTIMATE OF PARAMETERS - TM

α_1	α_2	α_3	α_4	k
0.43	-0.23	0.41	0.03	0.33
(4.71)	(-2.29)	(4.72)	(0.77)	(2.8)
σ_z	σ_u	σ_ε		
0.0004	0.016	0.014		
(7.01)		(6.41)		

t-stat in parentheses.

3, is a kind of detrended GDP with quite large fluctuations. This measure has a positive impact on inflation, through the parameter k.

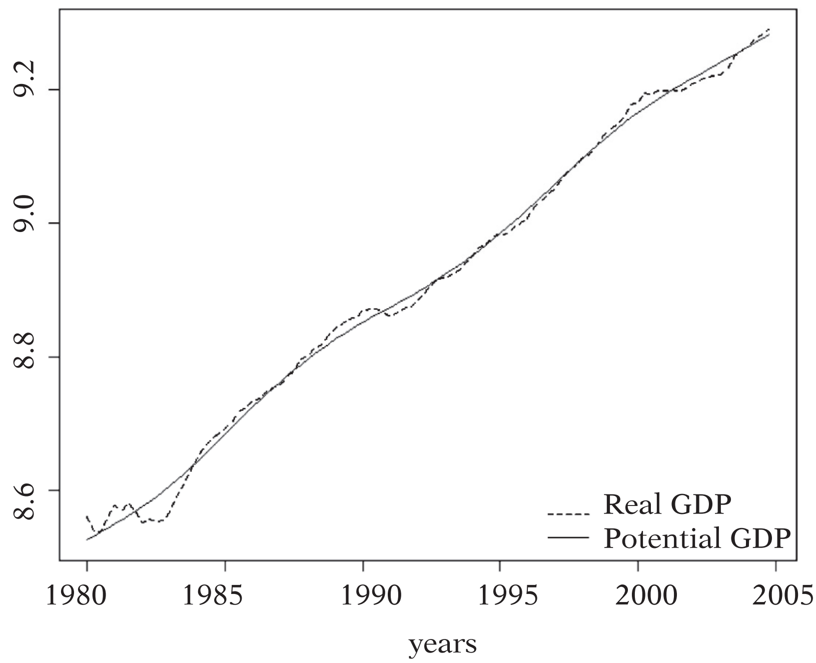
Another important feature about inflation is its considerable dependence from the past, as is clear from the coefficients α_i i = 1, 2, 3, 4. This peculiarity, commonly known as the persistence of inflation, is an evident characteristic of the data, and has to be taken into account in every model describing inflation.

4. - A New Keynesian Estimate of Output Gap and Inflation

In this section we propose a bivariate estimate labeled "New Keynesian". The potential output is coherent with Woodford's suggestion, and the New Keynesian model describes the dynamics of output gap and inflation. In the following analysis a basic small scale New Keynesian model is considered. It can be argued that medium scale versions of this model are more appropriate in empirical studies. However, the aim of this work is to stress the unsuitability of traditional measures of output gap in the New Keynesian framework, and to propose a theory-consistent measure estimated as a latent process. Unless we could obtain more accurate estimates using additional data, this would not affect the conclusion of this paper. Then, we chose the simplest alternative represented by a small scale model.

GRAPH 2

OBSERVED AND POTENTIAL OUTPUT - TM



GRAPH 3

OUTPUT GAP - TM

4.1 *A Basic New Keynesian Model*

The New Keynesian model, well described by Walsh (2003) among others, considers a microfounded framework of optimizing behavior. One of the key assumptions is the introduction of price rigidities so that, at time t, only a fraction $(1-\vartheta)$ (with $0 \leq \vartheta \leq 1$) can change the price and place it at its optimal level. Under these assumptions, we define the natural level of output, Y^n, as the output produced when prices are completely flexible, that is, when $\vartheta = 0$. The definition of output gap, as the natural level of output, is now more precise: $x_t = \tilde{y}_t - \tilde{y}_t^n$ where y_t is, as usual, the logarithm of the observed output, y_t^n is the logarithm of the natural output and the symbol ~ denotes the deviation of a variable from its steady state value. The term $\tilde{y}_t$ is called, in the literature, the «stochastic component of output»: the difference between the logarithm of output and its trend. Denoting with A_t an aggregate productivity disturbance, we can derive the following equation for the log deviation of the natural level of output:

$$\tilde{y}_t^n = \frac{1+\eta}{\phi+\eta}\tilde{\alpha}_t \tag{2}$$

where ϕ and η are the inverse of intertemporal elasticity of substitution, respectively in consumption and working hours. Finally, we use the definition of output gap to write the identity:

$$y_t = trend_t + \tilde{y}_t^n + x_t$$

and defining the potential output $y_t^p = trend_t + \tilde{y}_t^n$ we come back to the usual relation $y_t = y_t^p + x_t$ of system (*1*). Woodford's suggestion of treating y_t^p as a non-smooth measure is shared by many other authors (see, for example, Galì, 2002), and so we treat the potential output as in Model 3, allowing for shocks both to the level and to the growth rate. In particular, we use the hypothesis that $\tilde{\alpha}_t \sim N(0,\sigma_\alpha^2)$, so that $\tilde{y}_t^n = e_t \sim N(0,\sigma_e^2)$, with $\sigma_e^2 = \left(\frac{1+\eta}{\phi+\eta}\right)^2 \sigma_\alpha^2$. We suppose that e_t represents the only source of disturbance to the level of y_t^p.

The potential output is the long-term component of the output, while the short-term component is the output gap, the dynamics of which are described by the following “hybrid” New Keynesian IS curve (NKIS):

$$x_t = \xi E_t x_{t+1} - \frac{1}{\phi}(i_t - E_t \pi_{t+1}) + (1-\xi) x_{t-1} + u_t \qquad \text{(NKIS)}$$

The notation $E_t x_{t+1}$ stands for $E[x_{t+1} \mid I_t]$, that is, the expected value of x at t+1, given the information available at time t. In NKIS, we also have $u_t = E_t \tilde{y}^n_{t+1} - \tilde{y}^n_t$, and from equation *(2)* we have:

$$u_t \frac{1+\eta}{\phi+\eta}(E_t \tilde{\alpha}_{t+1} - \tilde{\alpha}_t) = -\tilde{y}^n_t$$

So, in our model, shocks to technology have two effects: they are sources of transitory fluctuations, as in Real Business Cycle models; and they permanently change the path of output. The cyclical fluctuations so generated are not trend reverting, coherent with the results of Nelson and Plosser (1982).

The dynamics of inflation are described by the New hybrid Phillips curve (NHPC) in Galì and Gertler (1999) who derived this equation using the hypothesis that a fraction ω of firms are backward looking, and they decide their prices considering only information until time t – 1. The remaining part (1 – ω) of the firms behaves as described by the New Keynesian model.

$$\pi_t = kx_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + g_t \qquad \text{(NHPC)}$$

Under Galì and Gertler’s hypothesis, the parameters γ_f, γ_b and k are so determined:

$$\gamma_f = \beta \vartheta \psi^{-1}$$
$$\gamma_b = \omega \psi^{-1}$$
$$k = (\phi+\eta)(1-\omega)(1-\vartheta)(1-\beta\vartheta)\psi^{-1}$$

where β is the subjective discount factor and $\psi = \vartheta + \omega[1 - \vartheta(1-\beta)]$.

In the New Keynesian IS another variable appears: the interest rate. Our intention, however, is to compare two bivariate systems, so we prefer to avoid, for the moment, to use further information: we simply substitute in NKIS a simple Taylor rule, supposing that $i_t = \mu E_t \pi_{t+1} + \upsilon_t$, with $\upsilon_t \sim N(0,\sigma_\upsilon^2)$.

Then, the version of the New Keynesian model we are going to estimate is the system (NKM):

$$\begin{aligned}
&y_t = y_t^p + x_t \\
&y_t^p = y_{t-1}^p + \delta_{t-1} + e_t \qquad e_t \sim N(0,\sigma_e^2) \\
&\delta_t = \delta_{t-1} + z_t \qquad z_t \sim N(0,\sigma_z^2) \\
&x_t = \xi E_t x_{t+1} + \frac{1-\mu}{\phi} E_t \pi_{t+1} + (1-\xi) x_{t-1} - e_t - \frac{1}{\phi}\upsilon_t \qquad \upsilon_t \sim N(0,\sigma_\upsilon^2) \\
&\pi_t = k x_t + \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + g_t \qquad g_t \sim N(0,\sigma_g^2)
\end{aligned} \tag{NKM}$$

NKM is a very interesting model, and has a peculiarity. Instead of finding, first, a measure of the output gap, detrending the observed output in some way, and, subsequently, estimating the system formed by the last two equations, we are trying to perform the two steps simultaneously. This opportunity is given by the DLM approach, which allows us to consider one or more series as the sum of more components. We proceed by solving the system with rational expectations (formed by the New Keynesian IS and the New hybrid Phillips curve) using Sims' method (see Sims, 2002), and, then estimate the DLM as the sum of the linear trend model and the rational expectations model. A similar approach is followed by Ferroni (2009).

In order to follow Sims' procedure, we first cast our rational expectations model in the canonical form:

$$\Gamma_0 W_t = \Gamma_1 W_{t-1} + \Psi \varepsilon_t + \Pi X_t$$

where ε_t is the vector with exogenous shocks and X_t which contains the expectation errors, satisfies $E_t X_{t+1} = 0 \ \forall t$. To make this step, remember the definition of rational expectation and express $x_t = E_{t-1} x_t + X_t^x$ and $\pi_t = E_{t-1} \pi_t + X_t^\pi$. Then, defining $W_t = (x_t \ \ \pi_t$

$E_t x_{t+1}\ E_t\pi_{t+1})'$, $\varepsilon_t = (e_t\ \ \upsilon_t\ \ g_t)'$ and $X_t = (X_t^x\ \ X_t^\pi)'$, we rewrite the model as:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & -\xi & \frac{\mu-1}{\phi} \\ -k & 1 & 0 & -\gamma_f \end{pmatrix} W_t = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1-\xi & 0 & 0 & 0 \\ 0 & \gamma_b & 0 & 0 \end{pmatrix} W_{t-1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -\frac{1}{\phi} & 0 \\ 0 & 0 & 1 \end{pmatrix} \varepsilon_t + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} X_t$$

The solution takes the form:

$$W_t = \Theta_1 W_{t-1} + \Theta_2 \varepsilon_t$$

where Θ_1 is a (4×4) matrix, and Θ_2 is a (4×3) matrix. So, the state space representation of (NKM) has the observation equation:

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_t^p \\ \delta_t \\ x_t \\ \pi_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} + \begin{pmatrix} 0 \\ g_t \end{pmatrix}$$

and the state equation:

$$\begin{pmatrix} y_t^p \\ \delta_t \\ x_t \\ \pi_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = \begin{pmatrix} \multicolumn{2}{c}{G_L} & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ 0 & 0 & & & & \\ 0 & 0 & & \Theta_1 & & \\ 0 & 0 & & & & \\ 0 & 0 & & & & \end{pmatrix} \begin{pmatrix} y_{t-1}^p \\ \delta_{t-1} \\ x_{t-1} \\ \pi_{t-1} \\ E_{t-1} x_t \\ E_{t-1} \pi_t \end{pmatrix} + \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & & & \\ 0 & & \Theta_2 & \\ 0 & & & \\ 0 & & & \end{pmatrix} \begin{pmatrix} z_t \\ e_t \\ \upsilon_t \\ g_t \end{pmatrix}$$

with $G_L = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ modeling the trend component.

4.2 *Empirical Evidence*

NKPC does not have any constant, but the mean of the observed process is different from zero, and the New Keynesian Phillips curve is derived loglinearizing the inflation around the zero steady state. Then, we choose to analyze the series obtained, subtracting from the inflation, its mean.

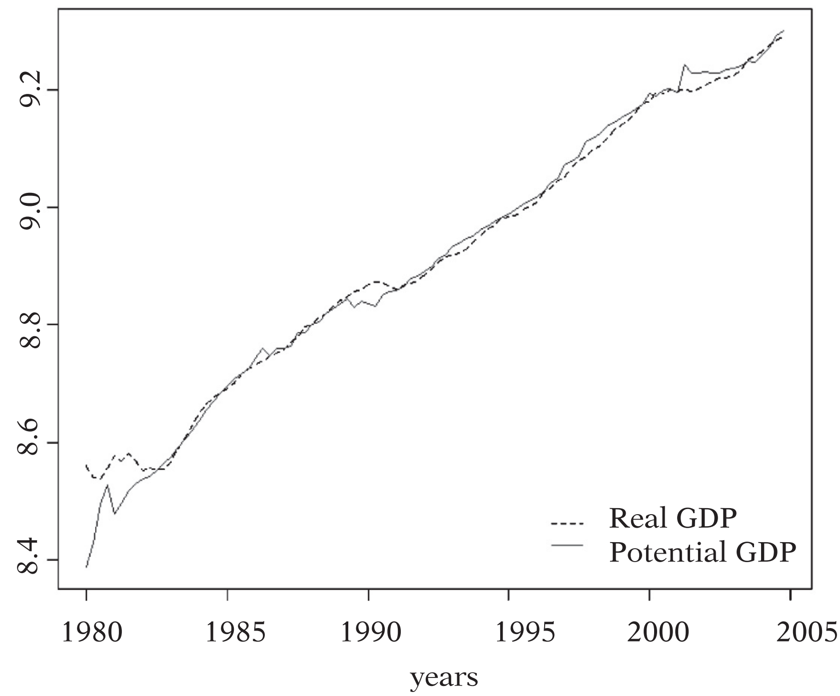
To reduce the problems of parameters' identification, we impose some restrictions: we set both elasticities, ϕ and η, equal to one. This assumption is quite standard in the literature. Another restriction is imposed to β, also set equal to one. This is not a serious misspecification: estimates found in the literature are always close to one. Moreover, this is a particular case of NHPC, also analyzed in Galì and Gertler (1999), in which the sum of γ_f and γ_b is exactly one. Another important restriction regards the parameter μ, imposed as greater than one: the Taylor principle is respected and the system with rational expectations has a unique solution.

Table 4 reports the estimates of the eight parameters. The comparison between permanent shocks to the output is similar to the one obtained in Model 3: disturbances to the growth rate are less important in explaining the variations of output. There are three sources of transitory movements (the output gap): the expectations of future inflation, the shocks to productivity (as previously mentioned), and the shocks expressed by the term υ_t. Note that ideally these disturbances represent deviations from the monetary policy rule; however, we don't use data about the interest rate, so υ_t contains all the other transitory shocks. The measure of potential output estimated with this model (Graph 4) is, as expected, clearly irregular. Graph 5 shows both the output gap obtained with the Kalman filter, and the output gap obtained with the Kalman smoother. As we will see in next Section, these series of output gaps are very different from the ones implied by the traditional model.

Also, the results concerning the NKPC are interesting. The value of the coefficients γ_f is such that a pure forward looking curve is estimated. Moreover, the value of k is positive and significant, in line with the theory's prediction.

GRAPH 4

OBSERVED AND POTENTIAL OUTPUT - NKM



GRAPH 5

OUTPUT GAP - NKM

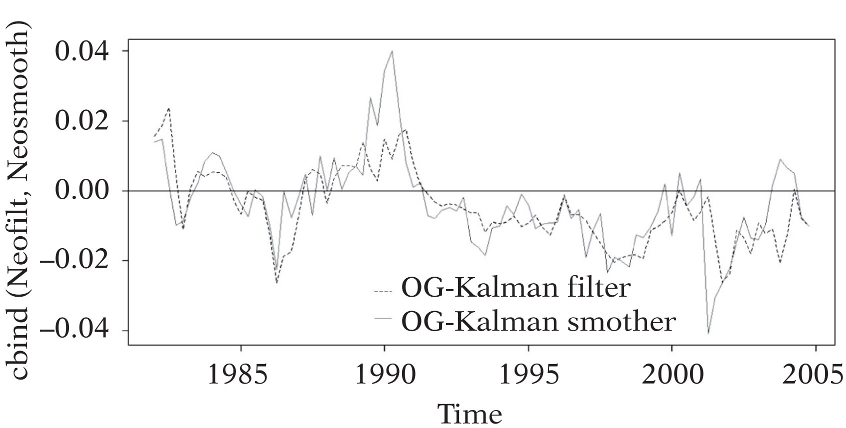


TABLE 4

ESTIMATE OF PARAMETERS - NKM

γ_f	ξ	μ	k
0.97	0.54	1.01	0.15
(52.37)	(34.16)	(2.32)	(2.03)
σ_e	σ_z	σ_v	σ_g
0.005	0.003	0.002	0.014
(1.71)	(2.45)	(1.25)	(5.76)

t-stat in parentheses

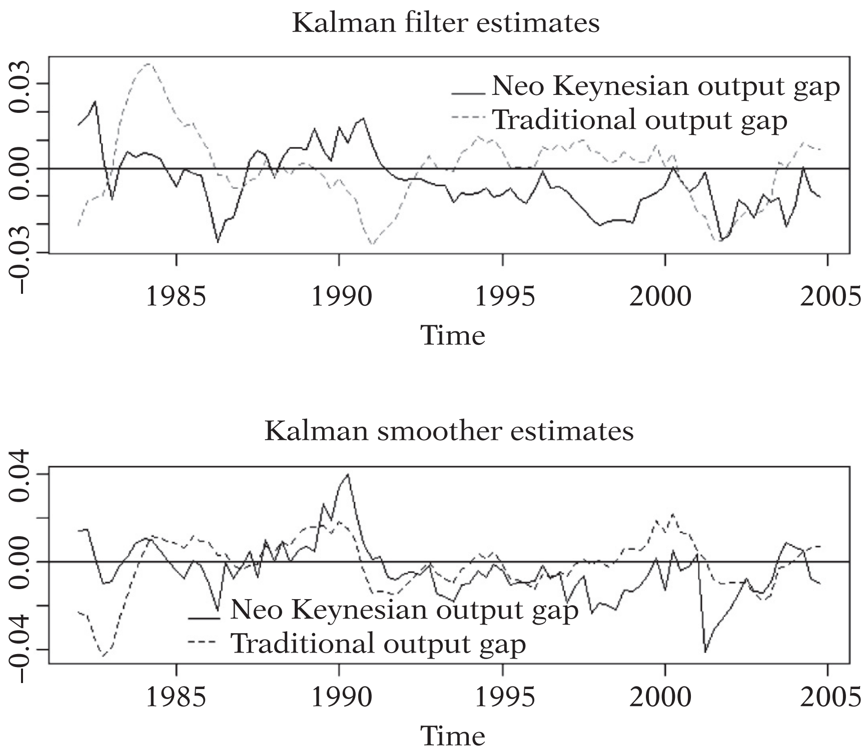
5. - Comparing the Traditional and the New Keynesian Output Gap

The famous HP filter used in (TM) is a particular case of Model 2, so it uses the hypothesis that output is $I(2)$. Then, the choice of the Model 3 specification in (NKM) can be better understood: the comparison of the two bivariate models is not affected by different assumptions about the integration order of the series of output. Moreover, note that, in both cases, σ_z is significantly different from zero.

Let us consider a simple qualitative analysis of the two couples of output gap series, shown in Graph 6. Look first at the filter estimates: even if the fluctuations are approximately the same size, the two series do not seem to comove. And if a certain comovement can be noted for the smoothed series, we stress that, in general, the traditional and the New Keynesian output gaps never meet at zero, and the New Keynesian series appears translated a bit to the right. This is consistent with the output gap measures generated by Neiss and Nelson (2002) that state: «the theory based output gap tends to lag detrended output». These last features depend on the correlation between inflation and output gap. What we expect in theory, in fact, from the «old» PC, is that output leads inflation. This means that a rise (or a decrease) in output should signal a rise (or a decrease) in inflation. In other words $corr(x_t, \pi_{t+1}) > 0$. The relation implied by the

GRAPH 6

COMPARING THE TRADITIONAL AND THE NEW KEYNESIAN OUTPUT GAP

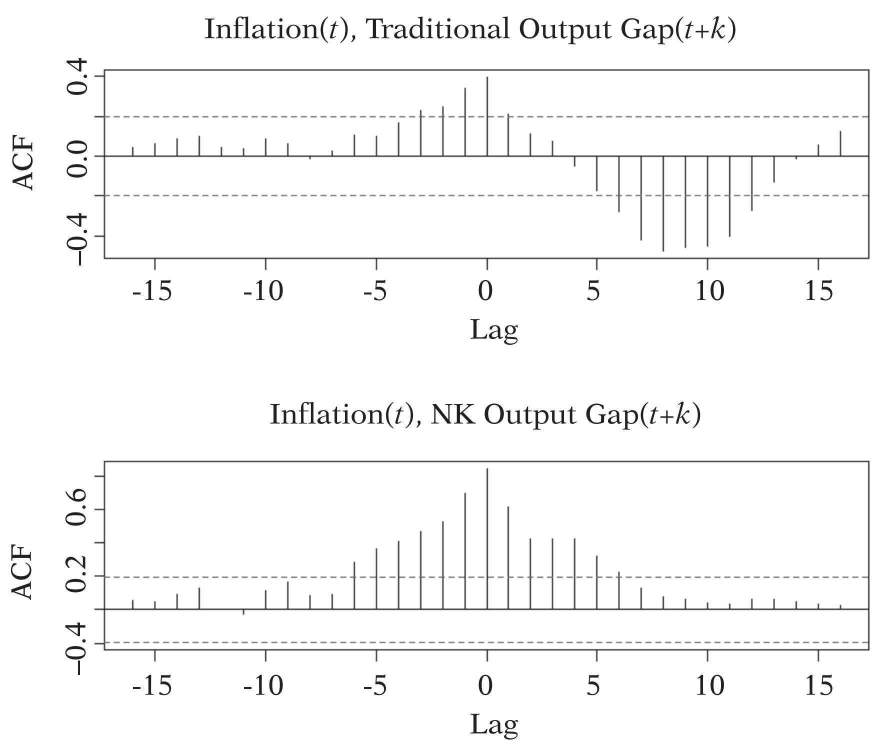


New Keynesian model is the opposite: inflation should lead output, or $corr(\pi_t, x_{t+1}) > 0$.

Galì and Gerlter (1999) use this argument to demonstrate the inability of the output gap in representing a proxy of the marginal costs. Calculating the cross correlation between the detrended GDP and inflation, and between the marginal costs and inflation, they verify that output leads inflation, while inflation leads marginal costs. However, in this work, we are trying to demonstrate that if we consider potential output to be a linear trend or a smooth measure, we are not estimating the output gap in the correct manner (in the New Keynesian framework). Consider, in fact, the first panel of Graph 7. It illustrates the cross correlation between the inflation rate and the output gap found by the traditional model (we consider the series obtained with the Kalman smoother). The pattern is absolutely similar to the one found by Galì and Gerlter (1999): in that case *output leads inflation*. The inconsistency in the New Keynesian theory emanates from the negative correlation between present inflation and future output gap. But if you consider the output gap estimated by the New Keynesian model, you arrive at the opposite conclusion. The second panel shows the cross correlation between this output gap and inflation: there is a sort of symmetry in the cross correlation, but the important feature is the positive correlation between present in-

GRAPH 7

CROSS CORRELATIONS



flation and future output gaps. In this sense we say that *inflation leads output*. The pattern is quite similar to the one found by Galì and Gertler (1999) for marginal costs and inflation. We conclude that if the output gap is estimated consistently with the New Keynesian model, a series with theory-consistent features can be found, and, therefore, an appropriate variable to be used in the New Keynesian Phillips curve.

6. - Conclusion

Traditional measures of potential output and of output gap are often criticized in works concerning New Keynesian models. For example, the bad performance of the New Keynesian Phillips curve in interpreting the data is linked to the inability of traditional output gap measures in driving the inflation rate. It is argued that one problem, stressed by Woodford and many other authors, is that potential output, in contrast with the New Keynesian model, is traditionally estimated as a smooth measure. Moreover, Galì and Gertler (1999) suggest that the output gap does not vary proportionally with marginal costs. Estimating a hybrid Phillips curve with the marginal costs instead of the output gap, they obtain better results. However, since inflation and output gap are both used in the Taylor rule, we could reasonably look for a correct measure of the output gap, and cure less about marginal costs. Our alternative would be to estimate the output gap consistently with the New Keynesian model. What we found is a measure that satisfies the predictions of the theory. In particular, considering what we called the New Keynesian output gap, we found that inflation leads output. Exactly the opposite relation is found if we utilize the traditional output gap.

Our intention is similar that of Neiss and Nelson (2002), but while in their work the output gap is generated using additional variables, in our case we estimate a latent process that is both consistent with the model and with the data. In other words we made an inference about the potential output and the output gap.

And we did this by considering only two variables: the (observed) output and the inflation rate.

This work has attempted to study the difference between permanent and transitory movements in the series of output. A possible improvement would consist in evaluating the same distinction for the series of inflation. In this model, inflation is loglinearized around a zero steady state, but we could presume a positive trend inflation, and consider the observed inflation as the sum of a trend component and a transitory component. It would be convenient to consider trend inflation in making inference about potential output because, as demonstrated by Ascari (2004), the presence of trend inflation, among other things, modifies the equilibrium level of output. Considering a time varying trend inflation would also allow us to consider a longer period, with bigger recessions, and more could be learned about the differences between traditional and New Keynesian measures of output gap.

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